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# Investment and Concern for Relative Position

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**Abstract**

Economists typically analyze individuals' market behavior in isolation from their nonmarket decisions. While this research strategy has generally been successful, it can lead to systematic errors when agents' nonmarket behavior affects their market choices. In this paper we analyze how individuals' investment behavior changes as a result of nonmarket behavior. Specifically, we analyze a model in which individuals must decide how to allocate their initial endowment between two random investments, where the returns are perfectly correlated across individuals for the first investment but independent across individuals for the second. We consider an environment in which men and women match, with wealthier individuals more successful in matching. We show how individuals' concern about relative wealth can affect their investment decisions, and we provide conditions under which individuals bias their investments either toward or away from the investment with correlated returns. A modification of the model is used to explain why agents investments might exhibit a home country bias.

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## 1. Introduction

The analysis of markets and consumer behavior in markets has been a central concern—perhaps *the* central concern—of economists since Adam Smith. While economists well understand that there are many important aspects of peoples' lives that are determined outside those markets, they investigate market behavior in the hope and belief that individuals' behavior in those markets can be studied, understood, or predicted isolated from the nonmarket behavior of those individuals. This separation of human activity into market and nonmarket components has, by and large, been hugely successful. The focus on market behavior has resulted in parsimonious general models of individual decision making that are applicable to a broad range of problems, rather than idiosyncratic, problem specific models.

While there are sound reasons underlying the modelling choice that segregates market from nonmarket behavior, that separation potentially comes at a large cost if individuals' decisions outside markets truly affect their attitudes and choices in markets. In earlier work, Cole, Mailath and Postlewaite (1992), we demonstrated how a particular decision that was not mediated through markets, matching between men and women, could affect market behavior.[1] In particular, we showed that if matching between men and women is positively assortative in wealth, matching considerations give people an incentive to save more than they would in the absence of matching concerns.

In this paper we investigate how nonmarket concerns, such as matching, can affect not only the level of investments that people make, but also the composition of their investments. Our interest is in how, in a given society, a particular agent's asset allocation decision is influenced by the asset allocations of others in that society. We are particularly interested in whether nonmarket activities might provide agents with an incentive to allocate assets in a manner similar to that of other agents in the society.

We analyze a model in which individuals allocate their initial endowment between two random investments, where the returns are perfectly correlated across individuals for the first investment but independent across individuals for the second. This standard portfolio choice problem is augmented with a matching/marriage decision following the realization of the agents' investments. It is assumed that following matching, consumption is joint, so that, all else equal, both men and women prefer wealthy partners to poor ones. The addition of a

[1] There is a substantial body of literature analyzing the interaction between market resource allocation mechanisms and nonmarket resource allocation mechanisms. See, e.g., Ichiishi (1993) and the references therein.

matching decision introduces a tournament aspect to the investment decision. The consumption of an agent depends not only on the realization of that agent's portfolio, but also on the wealth of that agent's partner in the subsequent matching. But an agent's ability to attract a wealthy partner depends not only on his or her own wealth, but also on the wealth of the "competition," the other agents of the same sex.

Hence, the addition of a matching decision to an otherwise standard portfolio choice problem adds an important general equilibrium component to the problem. In the absence of the nonmarket decision, each agent could make his or her investment decisions in isolation, since other agents's decisions have no effect on the agent's ultimate consumption; the addition of the particular nonmarket decision will generally make the agent's investment choice depend on other agents' choices. We show that the inclusion of the matching decision causes agents to care about the extent to which their wealth is correlated to that of other agents, and will provide conditions under which agents want to increase and decrease, relative to the standard model, that correlation.

The next section contains our formal model and results. In Section 3, we apply the model to a specific question of independent interest, why agents may fail to adequately diversify their investments. We use a variant of the model from Section 2 to show that if some agents in a group are exogenously constrained so as to prevent them from optimally diversifying, concern for relative rank will induce all agents to inadequately diversify, even though they may not be constrained. We conclude in Section 4 with a discussion and interpretation of our results, and a discussion of related literature.

## 2. The Investment Model

There is a continuum of men and a continuum of women, each uniformly distributed on $[0,1]$.[2] Male $i \in [0,1]$ is endowed with $e(i)$ units of the male good, where $e : [0,1] \to \Re_+$ is strictly increasing. In addition to this good, there is a nonconsumable capital good and a produced female consumption good. For simplicity, we assume that only females have the capital good, and hence the ability to produce the female consumption good. Females are *ex ante* identical, and each female is endowed with one unit of the capital good, which she can invest in either of two investment projects. There is a common project for which the random rate of return is identical across females and is denoted by $a$. The second project is

[2] The assumption that there is a continuum of men and a continuum of women is not innocuous. Making the number of agents finite would introduce subtleties absent from the continuum model; this is discussed in the last Section.

a private value project on which the rate of return is individual specific and will be denoted $b_i$; we assume $a$ and $b_i$ take on values between zero and one. We also assume that $a$ and each $b_i$ are independently distributed, with the distribution functions over the first and second investments given by $H$ and $F$ respectively, and the density functions denoted $h$ and $f$. The wealth of female agent $i$ is given by $y_i = x_i a + (1 - x_i) b_i$, where $x_i$ denotes the fraction of her capital invested in the common project. The female agents make their investment decisions prior to the realization of the rates of return on the two projects.

Females' preferences are given by $E\{u(\tilde{c}) + \hat{w}(\tilde{e})\}$, where $\tilde{c}$ denotes the (random) consumption level of the female good and $\tilde{e}$ denotes the (random) consumption of the male good. Since consumption of the male good is joint, it is determined by the identity of the male that the female matches with. We denote by $w$ the von Neumann utility of matching with male $j$, that is, $w(j) = \hat{w}\,(e(j))$. It is important to keep in mind that since the utility function over male mates $w$ is the composition of the endowment function $e$ and the utility function over male good $\hat{w}$, even if the utility function over the male good is concave, the endowment function $e$ may be such that the composition is either convex or concave (or neither). Males preferences are increasing in the consumption of both the male good (which is predetermined) and the female good. As will become clear, it is unnecessary to be more specific about male preferences.

The sequence of events is as follows. First, each female chooses an allocation of her capital between the two investment projects. After the returns of the projects are realized, males and females match. After matching, agents jointly consume the goods available to the pair. We assume that there is no commitment technology for enforcing agreements, which rules out any sort of insurance market among the females. Hence the only decisions that the agents make are their matching decisions and the females' investment decisions. Since all males value female wealth, and all females value male wealth (as measured by $e$), the only stable matching (in the sense of Roth and Sotomayer (1990)) is positively assortative in wealth. This allows us to restrict attention to positively assortative matchings, without specifying the matching process.

We restrict attention to symmetric equilibria, which substantially simplifies the analysis since if all females make identical investment decisions, we can assume that the distribution of females' wealth depends only on this common decision and the realized aggregate rate of return, $a$. It is relatively straightforward to determine the equilibrium matching outcome in this case. Since matching is positively assortative in wealth, a portfolio allocation $x \in [0, 1]$ is *a symmetric equilibrium* if each female finds it optimal to choose that portfolio allocation when all other females do so. Note that, apart from $x_i = x = 1$, the probability that female $i$ assigns to another female having the same final wealth as her is zero, irrespective of her portfolio decision. Thus, positive assortative matching on wealth determines her utility for all portfolio decisions. If $x_i = x = 1$ then all females are only investing in the common project, and in that case we assume each female is randomly matched with a male.

A common investment level $x$ for all women results in a determinate distribution of female wealth and hence a determinate matching conditional on an individual female's wealth. An equilibrium is characterized by a common level $x$ that maximizes each woman's expected utility given this matching relation.

First we calculate the distribution of female wealth that will result from a common division of capital between the two projects. This distribution will determine the probability distribution over matches that results from an arbitrary female's investment decision.

Assume that $x_{i'} = x$ for all $i' \neq i$. For a given realization $a$, female wealth within the population is distributed on $[xa, xa + (1-x)]$ with a fraction $F\left((y - xa)/(1-x)\right)$ having wealth less than or equal to $y$. Since the rank, $\tilde{s}_i$, of female $i$ with wealth $\tilde{y}_i$ is the fraction of females with wealth less than or equal to $\tilde{y}_i$,

$$\tilde{s}_i = F\left((\tilde{y}_i - xa)/(1-x)\right).$$

The probability, *conditional on* $a$, of her rank being less than or equal to $s_i < 1$ is given by[3]

$$\begin{aligned}
\Pr\{\tilde{s}_i \leq s_i | a\} &= \Pr\{F\left((\tilde{y}_i - xa)/(1-x)\right) \leq s_i | a\} \\
&= \Pr\left\{(\tilde{y}_i - xa)/(1-x) \leq F^{-1}(s_i) | a\right\} \\
&= \Pr\left\{\tilde{y}_i - xa \leq (1-x) F^{-1}(s_i) | a\right\}.
\end{aligned}$$

Female $i$ is investing $x_i$ in the common project, so $\tilde{y}_i = x_i a + (1-x_i) b_i$, and we have

$$\begin{aligned}
\Pr\{\tilde{s}_i \leq s_i | a\} &= \Pr\left\{x_i a + (1-x_i) b_i - xa \leq (1-x) F^{-1}(s_i) | a\right\} \\
&= \Pr\left\{ b_i \leq \frac{(1-x) F^{-1}(s_i) + (x - x_i) a}{(1-x_i)} \middle| a \right\} \\
&= F\left[\frac{(1-x)F^{-1}(s_i) + (x-x_i)a}{1-x_i}\right].
\end{aligned}$$

[3]The second equality does not hold for $s_i = 1$, since it is possible that $(\tilde{y}_i - xa)/(1-x) > 1$. While $F$ is only one-to-one on $[0,1]$, it is easy to see that the second equality does hold for $s_i = 0$.

Taking expectations over the returns of the common project (which is realized *after* female $i$ chooses $x_i$) yields the probability distribution function over female $i$'s rank contingent on her choice of $x_i$ alone, denoted $G(s_i;x_i)$,

$$G(s_i;x_i)=\int_0^1 F\left[\frac{(1-x)F^{-1}(s_i)+(x-x_i)a}{1-x_i}\right]h(a)da.$$

A female agent's rank in the output distribution determines her equilibrium match, and so her consumption of the male good, specifically, female $i$ of rank $s_i$ is matched with male $s_i$ and so receives utility $w(s_i)$. We now explicitly define the symmetric equilibrium portfolio decision.

**Definition 1.** *A portfolio allocation $x$ is* a symmetric equilibrium *if, for all $i$, $x_i=x$ solves*

$$\max \iint u(x_i a+(1-x_i)b_i)h(a)f(b_i)dadb_i+\int w(s_i)G(ds_i;x_i).$$

Consider for a moment the special case where both $u$ and $w$ are linear. In this case, females are both risk neutral and "rank neutral," that is, females care only about the expected rank in the wealth distribution. However, since rank is not a linear function of wealth, maximizing the expected value of one's wealth rank is not the same as maximizing expected wealth. This implies that there is a tradeoff between expected consumption and expected wealth rank. Our first observation is that if the distributions are symmetric, a female's expected wealth rank is independent of her portfolio choice.

**Lemma 1.** *Suppose the distributions of the returns on the projects are symmetric (i.e., $F(x)=1-F(1-x)$ and $H(x)=1-H(1-x)$ for all $x\in[0,1]$ ). Then a female's expected rank equals $\frac{1}{2}$, independent of her portfolio choice $x_i$.*

**Proof.** First, note that $F^{-1}(s)+F^{-1}(1-s)=1$. Now,

$$\begin{aligned}
G(1-s_i;x_i) &= \int_0^1 F\left[\frac{(1-x)F^{-1}(1-s_i)+(x-x_i)a}{1-x_i}\right]h(a)da \\
&= \int_0^1 F\left[\frac{(1-x)(1-F^{-1}(s_i))+(x-x_i)a}{1-x_i}\right]h(a)da \\
&= \int_0^1 \left\{1-F\left[1-\frac{(1-x)(1-F^{-1}(s_i))+(x-x_i)a}{1-x_i}\right]\right\}h(a)da \\
&= \int_0^1 \left\{1-F\left[\frac{(1-x)F^{-1}(s_i)+(x-x_i)(1-a)}{1-x_i}\right]\right\}h(a)da \\
&= 1-G(s_i;x_i),
\end{aligned}$$

where the last equality holds because $a$ and $1-a$ are identically distributed (i.e., $h(a) = h(1-a)$ ). Since $G(s_i; x_i)$ is symmetric, $\int s_i G(ds_i; x_i) = \frac{1}{2}$.

■

The first order condition associated with a female agent's problem is

$$\iint u'(x_i a + (1-x_i)b_i)\,(a-b_i)\,h(a)f(b_i)da db_i + \frac{\partial}{\partial x_i}\int w(s_i)G(ds_i; x_i) = 0.$$

Standard optimal portfolio arguments, which ignore matching effects, would imply that the first term in the above expression is equal to zero. To see how concerns about relative wealth can alter investment decisions, consider the following approximation to the second term:

$$\frac{\partial}{\partial x_i}\int w(s_i)G(ds_i; x_i) \approx w'(\frac{1}{2})\frac{\partial}{\partial x_i}\int (s_i - \frac{1}{2})G(ds_i; x_i) + \frac{1}{2}w''(\frac{1}{2})\frac{\partial}{\partial x_i}\int (s_i - \frac{1}{2})^2 G(ds_i; x_i).$$

This expression separates the second term into an expected rank component, and a variance of rank component.

If female $i$ invests $x_i$ in the first project, the support of the distribution of her possible output conditional on the realized value of $a$ is $[x_i a, x_i a + (1-x_i)]$ . If all other females invest $x$ in the first project, the support of the ex post distribution of female output conditional on $a$ is $[xa, xa + (1-x)]$. It follows therefore that if $x_i > x$, the support of female $i$'s conditional distribution of output lies strictly inside the support for the aggregate distribution for all $a \leq 1$. Similarly, if $x_i < x$ and $a \leq 1$, then while the expected level of female $i$'s output is still the same, the upper and lower supports of her conditional distribution exceeds those of the aggregate distribution. Thus, while female $i$ cannot affect her expected wealth rank because of the assumed symmetry, she may reduce the variance of rank by investing more heavily in the common project.

In the following example we illustrate how concerns over the variance of one's wealth rank can alter equilibrium outcomes. In the equilibrium of the example we consider, when agents have concave preferences over their rank they invest less in projects whose returns are idiosyncratic and more in projects whose returns are correlated with the returns of other agents than they would if these concerns were absent. If their preferences over their wealth rank are convex then the reverse is true.

**Example:** Assume that $u(c) = D_1 c - c^2$, $w(j) = d_1 j - dj^2$, and that $D_1$ and $d_1$ are both positive. The functions $u$ and $w$ will be increasing in the relevant range as long as $D_1 \geq 2$ and $d_1 \geq 2d$ (since $c, j \in [0,1]$). We also assume that each $b_i$ is uniformly distributed on $[0,1]$. We will assume that $a$ is symmetrically distributed on $[0,1]$, so that $\mathcal{E}a = \frac{1}{2}$. Given our quadratic preference specification only the first two moments of the distribution over $y_i$ and $s_i$ matter. Since both $a$ and $b_i$ are symmetrically distributed, then as shown above, the expected value of a female's status level is independent of her investment portfolio decision. As we will show below, the variance of a female agent's status is not independent of her investment decision. Similarly, since $\mathcal{E}y_i = x_i a + (1-x_i)b_i = \frac{1}{2}$, female $i$ can only affect the variance of her output, but not its expected value.

The variance of $y_i$, and hence of consumption, is given by

$$\mathcal{E}\left\{\left(x_i a + (1-x_i)b_i - \frac{1}{2}\right)^2\right\} = x_i^2\sigma_a^2 + (1-x_i)^2\,\sigma_b^2.$$

The choice of $x_i$ that minimizes the variance of $y_i$ thus solves

$$x_i\sigma_a^2 - (1-x_i)\sigma_b^2 = 0,$$

which implies

$$x_i = \frac{\sigma_b^2}{\sigma_a^2 + \sigma_b^2} \equiv x^*. \tag{1}$$

The consumption variance minimizing portfolio allocation, $x^*$, will play an important role in what follows.

The remainder of this section proves the following proposition. A symmetric equilibrium is *interior* if the equilibrium portfolio allocation has strictly positive weight on both the common project and the private value project.

**Proposition 1.** *Suppose* $u(c) = D_1 c - c^2$, $w(j) = d_1 j - dj^2$, *with* $D_1 \geq 2$, $d_1 \geq \max\{0, 2d\}$. *Assume the common project return distribution is symmetric and that the private project returns are uniformly distributed on* $[0,1]$. *If a symmetric equilibrium exists, it is unique. If a symmetric interior equilibrium exists, then* $4dx^* \leq (1-x^*)^2$ *and the fraction in the common project is given by*

$$x^- \equiv \frac{1 + x^* - \sqrt{(1-x^*)^2 - 4dx^*}}{2},$$

*which is a strictly increasing function of* $d$ *and equals* $x^*$ *when* $d = 0$. *Thus, if females are rank risk averse* ($d > 0$)*, they invest more than* $x^*$ *in the common project, while if they are rank risk loving* ($d < 0$)*, they invest less than* $x^*$ *in the common project.*

*For* $-1 \leq d \leq 0$, *a symmetric interior equilibrium exists if* $4dx^* \leq (1-x^*)^2$ *and* $(1-x^-)^2 \geq -d$.

*For $d \geq 0$, a symmetric interior equilibrium exists if $4dx^* \leq (1-x^*)^2$ and $(1-x^-)^2 \geq 2dx^*$.*

*For $d \leq -1$, a symmetric equilibrium exists and it is the boundary equilibrium at $x = 0$.*

*For $2dx^* \geq (1-x^*)^2$, a symmetric equilibrium exists and it is the boundary equilibrium at $x = 1$.*

Since females cannot affect either the expected value of their consumption or the expected value of their rank (Lemma 1), so we can ignore this component in their payoff. The relevant part of the expected payoff of female agent $i$ then has two parts: the variance of consumption and the variance of rank, which we denote by $A(x_i; x)$ and $B(x_i; x)$, respectively. The variance of consumption is

$$A(x_i; x) = \mathcal{E}(x_i a + (1-x_i)b_i - 1/2)^2 = x_i^2 \sigma_a^2 + (1-x_i)^2 \sigma_b^2.$$

**Lemma 2.** *The variance of rank is given by*

$$B(x_i; x) = \begin{cases} \frac{(x_i - x)^2 \sigma_a^2 + (1-x_i)^2 \sigma_b^2}{(1-x)^2}, & \text{if} \quad x_i \geq x, \\ \\ \frac{1}{4} - \frac{(1-x)}{6(1-x_i)}, & \text{if} \quad x_i < x. \end{cases}$$

**Proof.** Consider first $x_i \geq x$. In this case, the support of female $i$'s wealth is contained within the support of the aggregate distribution of female wealth. Conditional on $a$, the rank of female $i$ is given by $\{(x_i - x)a + (1-x_i)b_i\}/(1-x)$ for all $a$ and $b_i$. The variance of the rank is then (recall that $a$ and $b_i$ are independent)

$$\mathcal{E}\left\{\left[\frac{(x_i - x)a + (1-x_i)b_i}{1-x} - \frac{1}{2}\right]^2\right\} = \frac{(x_i - x)^2 \sigma_a^2 + (1-x_i)^2 \sigma_b^2}{(1-x)^2}. \tag{2}$$

Consider now $x_i < x$. Now, the distribution of possible output levels, conditional on $a$, does not lie within the support of the aggregate output distribution, and the ranks 0 and 1 occur with positive probability. Denoting female $i$'s rank by $s_i(b_i; a)$, we have

$$s_i(b_i; a) = \begin{cases} 0, & \text{if } b_i \leq (x - x_i)a/(1-x_i), \\ \left[(x_i - x)a + (1-x_i)b_i\right]/(1-x), & \text{if } (x - x_i)a/(1-x_i) < b_i \\ & \quad < \left[(x - x_i)a + (1-x)\right]/(1-x_i), \\ 1, & \text{if } \left[(x - x_i)a + (1-x)\right]/(1-x_i) \leq b_i. \end{cases}$$

Thus, with probability $(x - x_i)a/(1-x_i)$, female $i$ has wealth rank 0, with probability $(x - x_i)(1-a)/(1-x_i)$, rank 1, and with the residual probability, $(1-x)/(1-x_i)$, her rank is uniformly distributed between 0 and 1. The variance of her rank is $\mathcal{E}\left\{(s_i(b_i;a)-\frac{1}{2})^2\right\} = \mathcal{E}_a\mathcal{E}_b\left[(s_i(b_i;a)-\frac{1}{2})^2|a\right]$. We first evaluate the expectation conditional on $a$:

$$\begin{aligned}
\mathcal{E}_b\left[(s_i(b_i;a)-\frac{1}{2})^2|a\right] &= \frac{1}{4}\times\Pr\{s_i=0\}+\frac{1}{4}\times\Pr\{s_i=1\}+\\
&\quad \mathcal{E}_b[(s_i(b_i;a)-\frac{1}{2})^2|a, s_i\in(0,1)]\times\Pr(s_i\in(0,1))\\
&= \frac{(x-x_i)}{4(1-x_i)}+\int_{s^{-1}(0)}^{s^{-1}(1)}(s_i(b_i;a)-\frac{1}{2})^2\,db_i\\
&= \frac{(x-x_i)}{4(1-x_i)}+\int_0^1(s_i-\frac{1}{2})^2\left(\frac{ds_i}{db_i}\right)^{-1}ds_i\\
&= \frac{(x-x_i)}{4(1-x_i)}+\frac{(1-x)}{(1-x_i)}\int_0^1(s_i-\frac{1}{2})^2\,ds_i\\
&= \frac{(x-x_i)}{4(1-x_i)}+\frac{(1-x)}{12\,(1-x_i)}=\frac{1}{4}-\frac{(1-x)}{6\,(1-x_i)}. \qquad (3)
\end{aligned}$$

Since this is independent of $a$, this is also the unconditional variance of rank for $x_i \leq x$.

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The problem of female agent $i$ can be written as

$$\min_{x_i}\left[A(x_i;x)+dB(x_i;x)\right].$$

While $A(x_i;x)$ is obviously convex in $x_i$, $B(x_i;x)$ is more complicated (particularly for $x_i < x$), and may not be continuously differentiable in $x_i$ at $x$.

In the following, we first consider the right and left derivatives of the female's objective function. A necessary condition for an interior equilibria at $x$ is that the left derivative of the agent's objective function is nonpositive and the right derivative is nonnegative at $x_i = x$. We show that in this case, the two derivatives are equal and so equal zero, allowing us to solve for an interior equilibrium allocation when it exists. We then examine the relevant second order terms to establish conditions under which an interior equilibrium exists. Finally, we examine the conditions under which boundary equilibria might exist.

The derivative of $A(x_i;x)+ dB(x_i;x)$ with respect to $x_i$ for $x_i \geq x$ is

$$2\left[x_i+\frac{d(x_i-x)}{(1-x)^2}\right]\sigma_a^2-2\left[1+\frac{d}{(1-x)^2}\right](1-x_i)\sigma_b^2.$$

This is proportional to

$$2\left[x_i + \frac{d(x_i - x)}{(1-x)^2}\right](1-x^*) - 2\left[1 + \frac{d}{(1-x)^2}\right](1-x_i)x^*. \tag{4}$$

The derivative is nonnegative at $x_i = x$ only if

$$x - x^* - \frac{dx^*}{1-x} \geq 0. \tag{5}$$

The derivative of a female's objective function when $x_i < x$ is nonpositive only if

$$2\left[x_i(\sigma_a^2 + \sigma_b^2) - \sigma_b^2\right] + \frac{d(1-x)}{6(1-x_i)^2} \leq 0,$$

which, dividing by $(\sigma_a^2 + \sigma_b^2)2$, and making use of the fact that $\sigma_b^2 = 1/12$, yields

$$(x_i - x^*) - \frac{dx^*(1-x)}{(1-x_i)^2} \leq 0. \tag{6}$$

Evaluating at $x_i = x$,

$$x - x^* - \frac{dx^*}{1-x} \leq 0. \tag{7}$$

Since (5) and (7) must hold simultaneously for $x$ to be an interior equilibrium, the right and left hand derivatives are both equal to zero. This gives us a simple quadratic in $x$, which has two solutions

$$x^+ \equiv \frac{1 + x^* + \sqrt{(1-x^*)^2 - 4dx^*}}{2},$$

and

$$x^- \equiv \frac{1 + x^* - \sqrt{(1-x^*)^2 - 4dx^*}}{2}. \tag{8}$$

For the roots to be real, we need (as claimed)

$$4dx^* \leq (1-x^*)^2. \tag{9}$$

If $d < 0$, only $x^-$ is admissible, because $x^+$ is strictly larger than one. While both roots are feasible for some values of $d \geq 0$, we show below that only $x^-$ is consistent with the second order conditions. Note that $x^-$ is increasing in $d$; if $d = 0$, then $x^- = x^*$, while if $d > 0$, $x^- > x^*$ and if $d < 0$, $x^- < x^*$.

Additionally, for $x^-$ to be between zero and one,

$$1 + x^* - \sqrt{(1-x^*)^2 - 4dx^*} \leq 2,$$

which is implied by $x^* < 1$.

In order to verify that $x_i = x^-$ is a solution to the female $i$'s problem it is sufficient to show that her objective function is convex (recall that we have written her problem as a minimization problem). We find sufficient conditions for the derivatives in (4) and (6) to be both nondecreasing in $x_i$. The derivative of the expression in (4) with respect to $x_i$ is nonnegative if

$$1 + \frac{d}{(1-x)^2} \geq 0,$$

which, evaluated at $x = x^-$, is $(1-x^-)^2 \geq -d$. It is straightforward to verify that this holds with equality at $d = -1$ and is violated for all $d < -1$.

The derivative of the left hand side of (6) with respect to $x_i$ is nonnegative if

$$1 - \frac{2d(1-x)x^*}{(1-x_i)^3} \geq 0. \tag{10}$$

This condition is always satisfied if $d < 0$. If $d \geq 0$, a necessary and sufficient condition for (10) to hold for all $x_i \leq x$ is that it hold at $x_i = x$:

$$1 - \frac{2dx^*}{(1-x)^2} \geq 0. \tag{11}$$

Evaluating at $x = x^-$ yields $(1-x^-)^2 \geq 2dx^*$.

We now show why, for the case $d \geq 0$, the root $x^+$ is not a symmetric equilibrium. Evaluating (11) at $x = x^+$ and simplifying yields the inequality

$$(1-x^*)^2 - 4dx^* \geq 2(1-x^*)\sqrt{(1-x^*)^2 - 4dx^*} + 4dx^*,$$

which in turn requires

$$\sqrt{(1-x^*)^2 - 4dx^*} \geq 2(1-x^*),$$

or

$$-4dx^* \geq 3(1-x^*)^2,$$

which is impossible. But this implies that the female's objective function when $x = x^+$ is not quasiconvex in a neighborhood of $x_i = x = x^+$.

We turn now to the possibility of boundary equilibria. We consider first the case where $d > 0$. There cannot be a boundary equilibrium at $x = 0$, because inequality (5) would need to hold at $x = 0$, which is impossible when $d > 0$. If $x = 1$, and female $i$ sets $x_i = 1$, then all females have the same output. In this case the value of her objective function is $\sigma_a^2 + d/12$, since the male match quality is uniformly distributed on the unit interval and hence has variance $1/12$. If on the other hand, a female deviates by setting $x_i < 1$, then she receives the best match if $b_i > a$ or the worst match $b_i < a$. Since the event $a = b$ is a probability zero event, and $a$ and $b$ are symmetrically distributed around $\frac{1}{2}$, she will receive rank 1 or 0 with probability $\frac{1}{2}$. This implies that if a female sets $x_i < 1$,then the best she can do is set $x_i = x^*$, in which case the value of her objective function is $(x^*)^2 \sigma_a^2 + (1-x^*)^2\sigma_b^2 + d/4$. Thus, for there to be a boundary equilibrium at $x = 1$,we must have

$$(x^*)^2 \sigma_a^2 + (1-x^*)^2 \sigma_b^2 + d/4 \geq \sigma_a^2 + d/12,$$

or, rearranging,

$$d/6 \geq (1-(x^*)^2)\sigma_a^2 - (1-x^*)^2 \sigma_b^2.$$

But substituting for $\sigma_b^2 = \frac{1}{12}$ on the left hand side and dividing by $\left(\sigma_a^2 + \sigma_b^2\right)$ yields

$$\begin{aligned} 2dx^* &\geq \left(1-(x^*)^2\right)(1-x^*) - (1-x^*)^2 x^* \\ &= (1-x^*)^2, \end{aligned}$$

or

$$d \geq \frac{(1-x^*)^2}{2x^*} > 0. \tag{12}$$

It is worth noting that if $2dx^* < (1-x^*)^2 < 4dx^*$, then no symmetric equilibrium exists (since both (9) and (12) are violated.

Consider now $d < 0$. From (12), $x = 1$ cannot be a boundary equilibrium. The other possible boundary equilibrium is at $x = 0$. In this case, the distribution of aggregate output is uniform on the unit interval, and hence a female's relative wealth rank is equal to her output. In this case her objective function is given by (again from the expressions of $A(x_i; x)$ and $B(x_i; x)$)

$$(1+d)\left[x_i^2\sigma_a^2 + (1-x_i)^2\sigma_b^2\right].$$

This objective function is minimized at $x_i = 0$ if and only if $d \leq -1$; otherwise $x_i = x^*$, and there cannot be a boundary equilibria at $x_i = x = 0$.

## 3. Multiple common projects

As we discussed above, the model illustrates how a concern for relative position can lead agents to bias their decisions either toward or away from the average investment strategy of their reference group. We now consider an interesting application of this observation. Economists have for some time puzzled that individuals inadequately hedge their investments across countries (see, e.g. Lewis (1999)). The analysis of our model suggests a possible explanation. We show that if some agents are constrained to bias their portfolio (for example, rules that restrict institutions to invest only in home-country companies), then this will induce all other agents to also bias their portfolio in order to minimize the variance of their rank with respect to the constrained group.

To illustrate this point, we consider in this section a modification of the model above. We consider the case in which there are two common projects (which we could interpret as domestic and foreign market portfolios), with each project's returns distributed symmetrically and independently on the unit interval. All other details of the model are unchanged.

Suppose females are rank risk averse. We first argue that in any symmetric equilibrium, the portfolio allocation between the two common projects must minimize the variance of the common project portfolio returns. This implies that pure herding cannot explain portfolio bias. We then show that if some fraction of the population is forced to bias their portfolio, then this can induce the other agents to bias their portfolio in a similar direction.

**Proposition 2.** *Assume the common project returns are symmetrically and independently distributed on* $[0,1]$*, and that the private project returns are uniformly distributed on* $[0,1]$*. Suppose females are rank risk averse and that* $(x^1, x^2, 1-x^1-x^2)$ *is a symmetric equilibrium portfolio. Then,* $x^1/(x^1+x^2)$ *is the share in the first common project that minimizes the variance of returns from the common project portfolio.*

**Proof.** Suppose all females other than $i$ choose portfolio $(x^1, x^2, 1-x^1-x^2)$. Define $x \equiv x^1 + x^2$ and $\gamma \equiv x^1/x$. Since a female's optimal portfolio optimally allocates the investment in common projects between the two common projects, it is enough to argue that, for $\gamma_i \neq \gamma$, the rank of female $i$ under portfolio $(\gamma_i x, (1-\gamma_i)x, 1-x)$ is a mean preserving spread of her rank under portfolio $(\gamma x, (1-\gamma)x, 1-x)$.

We first describe the distribution of female $i$'s rank for arbitrary $\gamma_i$. These calculations are very similar to those for the case of a single common project. For given realizations of $a^1$and $a^2$, female wealth within the population is distributed on $[\gamma x a^1 + (1-\gamma) x a^2, \gamma x a^1 + (1-\gamma) x a^2 + (1-x)]$ with a fraction $F((y - \gamma x a^1 - (1-\gamma) x a^2)/(1-x))$ having wealth less than or equal to $y$. Since the rank, $\tilde{s}_i$, of female $i$ with wealth $\tilde{y}_i$ is the fraction of females with wealth less than or equal to $\tilde{y}_i$,

$$\tilde{s}_i = F\left(\left(\tilde{y}_i - \gamma x a^1 - (1-\gamma) x a^2\right) / (1-x)\right).$$

The probability, *conditional on* $a^1$ *and* $a^2$, of her rank being less than or equal to $s_i < 1$ is given by (since $F^{-1}(x) = x$ for $x \in [0,1]$)

$$\begin{aligned}
\Pr\{\tilde{s}_i \le s_i | a\} &= \Pr\left\{\tilde{y}_i - \gamma x a^1 - (1-\gamma) x a^2 \le (1-x) s_i | a^1, a^2\right\} \\
&= \Pr\left\{\tilde{y}_i - \gamma x \left(a^1 - a^2\right) - x a^2 \le (1-x) s_i | a^1, a^2\right\}.
\end{aligned}$$

Since female $i$ is also investing $x$ in the common projects, $\tilde{y}_i = \gamma_i x (a^1 - a^2) + x a^2 + (1-x) b_i$, and we have

$$\begin{aligned}
\Pr\{\tilde{s}_i \le s_i | a\} &= \Pr\{\gamma_i x \left(a^1 - a^2\right) + (1-x) b_i - \gamma x \left(a^1 - a^2\right) \le (1-x) s_i | a^1, a^2\} \\
&= \Pr\left\{ b_i \le s_i + \frac{(\gamma - \gamma_i) x (a^1 - a^2)}{(1-x)} \middle| a^1, a^2 \right\} \\
&= F\left[s_i + \frac{(\gamma - \gamma_i) x (a^1 - a^2)}{(1-x)}\right].
\end{aligned}$$

Taking expectations over the returns of the common projects yields the probability distribution function over female $i$'s rank contingent on her choice of $\gamma_i$ alone, denoted $G(s_i; \gamma_i)$,

$$G(s_i; \gamma_i) = \mathcal{E}_{a^1, a^2} F\left[s_i + \frac{(\gamma - \gamma_i) x (a^1 - a^2)}{(1-x)}\right].$$

The uniformity of $F$ and the symmetry of the distributions of $a^1$ and $a^2$ imply that $G$ is also symmetric:

$$\begin{aligned}
G(s_i; \gamma_i) &= \mathcal{E}_{a^1, a^2}\left\{1 - F\left[1 - s_i - \frac{(\gamma - \gamma_i) x (a^1 - a^2)}{(1-x)}\right]\right\} \\
&= \mathcal{E}_{a^1, a^2}\left\{1 - F\left[1 - s_i + \frac{(\gamma - \gamma_i) x (a^1 - a^2)}{(1-x)}\right]\right\} \\
&= 1 - G(1 - s_i; \gamma_i).
\end{aligned}$$

Not surprisingly, if $\gamma_i = \gamma$, the distribution of rank is uniform on $[0,1]$. We want to show that when $\gamma_i \ne \gamma$, the distribution of rank is a mean preserving spread of the uniform distribution. Since $G(\cdot; \gamma_i)$ is symmetric, we need only show that for any $s_i \in (0,1)$, the density of $G(\cdot;\gamma_i)$ never exceeds 1, and that it is strictly less than 1 on a positive measure set of $s_i$.

Let $z = a^1 - a^2$; we denote the density of $z$ by $k$. The random variable $z$ is distributed symmetrically on $[-1.1]$. Suppose $\zeta \equiv (\gamma - \gamma_i)\, x / (1-x) > 0$. Since $z$ is symmetrically distributed around 0, the following analysis also covers the possibility $\zeta < 0$. Fix $s_i$. We consider first the case $s_i < \zeta$ and $1 - s_i < \zeta$. The first inequality implies that for low realizations of $z$, $s_i + \zeta z < 0$ , while the second inequality implies that for large realizations of $z$, $s_i + \zeta z > 1$. Thus,

$$\begin{aligned} G(s_i;\gamma_i) &= \int_{-1}^{-s_i/\zeta} F(s_i+\zeta z)\, k(z)\, dz + \int_{-s_i/\zeta}^{(1-s_i)/\zeta} F(s_i+\zeta z)\, k(z)\, dz \\ &\quad + \int_{(1-s_i)/\zeta}^{1} F(s_i+\zeta z)\, k(z)\, dz \\ &= \int_{-s_i/\zeta}^{(1-s_i)/\zeta} (s_i+\zeta z)\, k(z)\, dz + \int_{(1-s_i)/\zeta}^{1} k(z)\, dz, \end{aligned}$$

since $F(s_i+\zeta z) = 0$ for $s_i + \zeta z \le 0$ and $F(s_i+\zeta z) = 1$ for $s_i + \zeta z \ge 1$. The density of $G$ is then (using Leibniz's rule)

$$g(s_i;\gamma_i) = \int_{-s_i/\zeta}^{(1-s_i)/\zeta} k(z)\, dz.$$

The other cases are handled analogously, so that we have

$$g(s_i;\gamma_i) = \begin{cases} \int_{-s_i/\zeta}^{(1-s_i)/\zeta} k(z)\, dz, & \text{if } s_i < \zeta \text{ and } 1-s_i < \zeta, \\ \int_{-1}^{(1-s_i)/\zeta} k(z)\, dz, & \text{if } s_i \ge \zeta \text{ and } 1-s_i < \zeta, \\ \int_{-s_i/\zeta}^{1} k(z)\, dz, & \text{if } s_i < \zeta \text{ and } 1-s_i \ge \zeta, \\ 1, & \text{if } s_i \ge \zeta \text{ and } 1-s_i \ge \zeta. \end{cases}$$

Note that $g(s_i;\gamma_i) < 1$ in all cases except for the last.

■

This result implies that the rank variance minimizing value of $\gamma_i$ equals $\gamma$. However, to the extent that this differs from the consumption variance minimizing level of $\gamma_i$, the optimal choice will involve trading off these costs. Since at the rank (consumption) variance minimizing value, the first-order effect of shifting $\gamma_i$ from this value is zero, the optimal choice will lie strictly between the rank and consumption variance minimizing values. Since this is true for all of the females, it implies that the only equilibrium value of $\gamma$ is the consumption variance minimizing value, which is 1/2 when the projects are symmetrically, independently and identically distributed.

We next show that if some fraction of the population is constrained to invest more than 1/2 of the resources that they devote to investing in the common projects in one of these projects, this induces other females to invest relatively more in this project as well.

**Proposition 3.** *If a fraction $\alpha > 0$ of the females is constrained to set $\gamma_i = \bar{\gamma} > 1/2$ , then in the symmetric equilibrium, all other females set $\gamma_i > 1/2$.*

Suppose the females in $[0, \alpha)$ are constrained, and that they choose portfolio $(\bar{\gamma}x, (1-\bar{\gamma})x, 1-x)$. Suppose all females other than $i$ in $[\alpha, 1]$ choose portfolio $(\gamma x, (1-\gamma)x, 1-x)$. As before, we first describe the distribution of female $i$'s rank for arbitrary $\gamma_i$. Here there are two different distributions of female returns. However, the rank of female $i$ will be a weighted average of her rank in each of these two populations, with the weights given by the respective fractions of the total population. Hence

$$\begin{aligned} s_i &= (1-\alpha)F\left(\left(\tilde{y}_i - \gamma x a^1 - (1-\gamma)\, x a^2\right) / (1-x)\right) + \\ &\quad \alpha F\left(\left(\tilde{y}_i - \bar{\gamma} x a^1 - (1-\bar{\gamma})\, x a^2\right) / (1-x)\right). \end{aligned}$$

This immediately implies that her expected rank is 1/2 regardless of her investment decision. This implies that the probability distribution function over female $i$'s rank contingent on her choice of $\gamma_i$ alone, which we again denote by $G(s_i; \gamma_i)$, is a weighted average of her probability distribution within each of these populations, or

$$\begin{aligned} G(s_i; \gamma_i) &= \mathcal{E}_{a^1,a^2}\left\{\alpha F\left[F^{-1}(s_i) + \frac{(\bar{\gamma} - \gamma_i)\, x\,(a^1 - a^2)}{(1-x)}\right]\right. \\ &\quad \left. + (1-\alpha)F\left[F^{-1}(s_i) + \frac{(\gamma - \gamma_i)\, x\,(a^1 - a^2)}{(1-x)}\right]\right\}. \end{aligned}$$

By the same logic as before, if female $i$ sets $\gamma_i = \gamma$, the variance of her rank among the population of females $[\alpha, 1]$ is minimized, and the first-order cost of deviating is zero. Hence, if $\gamma = 1/2$, then the first-order cost of deviating in the direction of $\bar{\gamma}$ is zero both in terms of the variance of her consumption and the variance of her rank among the $[\alpha, 1]$ females. Hence it would be optimal to do so. However, if it is optimal to do for her, it is optimal for all of the females $j \in [\alpha, 1]$. Hence, in equilibrium $\gamma > 1/2$. Moreover, since in equilibrium $\gamma_i = \gamma$, this implies that all the females in $[\alpha, 1]$ have minimized the variance of rank among the population $[\alpha, 1]$ and hence, at the margin, they are off increasing the variance of their consumption against decreasing the variance of their rank amongst the females in $[0, \alpha)$.

## 4. Discussion

1. In laying out the model, we mentioned that there could be subtle differences between a model with a continuum of men and a continuum of women and a model in which the number of agents were finite. In a model with a finite number of agents, there would typically be a positive distance between the wealth of any two women. Because of this, very small changes in any woman's wealth, up or down, will not affect the man with whom she was matched. Large changes, of course, will affect the wealth of her partner. Consequently, a woman will find attractive a lottery with a large probability of a sufficiently small loss and a small probability of a large gain. This phenomenon is illustrated by Robson (1996b), who shows in a biologically motivated model that finiteness can induce agents who are "fundamentally" risk neutral to choose some risky lotteries. Robson's model thus shares with our model the property that even when individuals are "fundamentally" risk neutral, their attitudes toward risk over wealth will depend on the shape of the function relating wealth to the items over which fundamental preferences are defined.[4] This also illustrates that our restriction to symmetric random variables is not without cost.
2. In addition to the work mentioned above, Robson has pursued a research agenda that is conceptually closely related to our approach. Robson (1992) lays out a model of decision making in which it is assumed that agents care not only about their wealth, but also about their relative standing in the wealth distribution (status). He assumes that individuals have identical utility functions that are concave in wealth, but convex in status. He uses this framework to show that even though an individual's utility function is concave in wealth itself, utility can nevertheless be convex in wealth over some ranges because of the indirect utility in status. He further investigates which income distributions would have the property that no individual would accept fair gambles.

As does our paper, that paper illustrates how a concern for relative standing can affect risk attitudes. However, it takes the concern for status, or relative rank, as exogenously given. The paper mentioned above, Robson (1996b), however, endogenizes the concern. In that model males care about maximizing expected number of offspring and females match with males to maximize resources per offspring. If a male has twice the resources as a second male, he will be able

[4]Robson (1996a) provides a nice discussion of this latter point.

to attract twice the number of females, and consequently, will have twice the expected number of offspring. Hence as in our model, males care about their relative wealth position not because it is exogenously posited, but because it affects equilibrium quantities.

While there is a close relationship between our work and that of Robson, there are also several substantial differences. Robson's focus is on the risk attitudes that would likely arise from evolutionary pressures due to biology. We begin from the conventional economic assumption of risk aversion over consumption, and are interested in how the inclusion of decisions that are not mediated by markets, such as matching, can alter the attitudes toward risky gambles over wealth. Our emphasis is not on explaining which risk attitudes will be hard-wired. Instead, we ask, taking as given whatever the hard-wiring of attitudes towards risk is, how do nonmarket mediated activities affect decision making? More generally, our work is distinguished from evolutionary models in that agents are fully rational, and we study equilibrium behavior. Agents understand fully the consequences of their actions and maximize utility. A last, more minor difference between this paper and Robson's is that the emphasis here is not so much on the attitudes towards risk *per se*, but on the degree to which individuals will want their situations correlated with other agents' situations.[5]

3. A consequence of agents desiring to increase the correlation of their portfolio to that of others is that we should see more "clustering" of agents than in a model devoid of incentives to mimic. Bikhchandani, Hirshleifer and Welch (1992) investigate what they term a striking regularity, localized conformity. They identify several mechanisms that have been suggested as explanations of a higher than expected degree of conformity within groups: sanctions on deviants, positive payoff externalities, conformity preference, and communication. Bikhchandani et al. (1992) present an alternative explanation—informational cascades—that they suggest explains not only conformity in groups, but also the fragility of the conformist behavior in the sense that the observed behavioral patterns can change swiftly and without obvious cause. The basic idea of informational cascades is as follows. Suppose that there is a sequence of similar agents who are making similar decisions in an uncertain common environment, and further, that the agents have independent signals about the state of the environment. When a particular individual is to make his decision, previous agents' decisions provide additional information in that they reflect the information the agents had at the time they made their decisions. Bikhchandani et. al. show that in some circumstances,

[5]Robson (1998) analyzes an evolutionary model in which there is a choice between idiosyncratic and common random variables. The choice there is driven by questions of observability of outcomes rather than competition.

agents may find it optimal to ignore their own private information and simply mimic the decisions taken by previous agents. In this event, agents will be seen making identical choices even though they have different information.[6]

When preferences over rank are concave, our model will also exhibit more conformity of behavior than would be expected if the nonmarket matching was ignored. As the example demonstrated, in this case agents invested more in projects whose returns are correlated with the returns of other agents than they would if there were no ranking concerns. Further, while our model isn't dynamic, it is clear that the underlying idea behind the conformity of behavior suggests fragility: if other agents in my reference group change investment strategies for some reason, I have an incentive to follow their lead to avoid increasing my risk of rank. The driving force however, is clearly different from that underlying informational cascades. If informational cascades arise, it is because agents are asymmetrically informed, while there is no asymmetry of information in our model. The conformity of behavior in our model is driven entirely by an *induced* desire to be similar to other agents. We emphasize "induced" because there is no inherent desire to conform in the agents' "deep" preferences; those preferences depend only on consumption.[7] Any desire to conform comes entirely from the nonmarket decisions—matching—and the consumption changes that follow more or less conformity with other agents' investment decisions.

5. The role matching plays in our model is to induce naturally a concern for relative position in the wealth distribution. While matching appears to us a plausible reason that individuals would care about relative rank, it is by no means the only such reason. Technically, all that is necessary for our results is the existence of a benefit or reward that accrues to each agent strictly as a function of the rank in the wealth distribution. If there are any positive consequences that come from simply being wealthier than others in the reference group, the analysis would be essentially the same as above. One can imagine any number of things richer agents might enjoy, such as preferential treatment for oneself or one's family in restaurants, schools, hospitals, churches or synagogues, easier access to politicians or governmental officials, and so on. All that matters is that the better treatment is not paid for directly, but rather is tied to one's rank in the wealth distribution.

There is an alternative interpretation of our model and results. Suppose one took the view that evolution has hard-wired into humans a concern for rank as in Robson (1992). In our model, women get indirect utility from rank equal to

[6] See also Banerjee (1992).

[7] This distinguishes the basis for conformism in our model from that in models such as Bernheim's Bernheim (1994) that assume a direct taste for conformist behavior.

the utility they would get from the wealth of the man at that rank level. If one posited that women got direct utility from rank equal to this utility, there would be no change in the analysis. The discussion would be about whether the utility function over rank was concave or convex rather than whether the male wealth function was concave or convex, but except for terminology, it would be the same.

6. The previous two discussion points suggest that there can be substantial differences in the degree of conformity of investment between different groups. As noted above, even if all agents have identical preferences over consumption, preferences over rank will differ across groups to the extent that the rewards to rank differ across those groups. In particular, if an agent's reward to rank was precisely the wealth of the person with whom that agent was subsequently matched, differences in the wealth distribution of potential mates would lead to different induced preferences over rank. For example, if in one group the wealth distribution of potential mates was convex, and in a second, concave, we would expect to see relative high amounts of correlation among investments in the first but high amounts of idiosyncratic investments in the second. Aside from differences of concavity/convexity, if the wealth distribution of potential mates in one group is twice that of a second group, both groups will exhibit the same bias toward or against correlated investments, but the bias will be greater in the group with wealthier potential mates. These implications hold not only across groups, but within a group across time. That is, if the wealth distribution among potential mates increases across time, any bias toward or against correlation of investments will be exacerbated.

In the point above, we argued that there can be sources other than the wealth of potential mates for a concern for rank. We would expect that the differences among groups in these kinds of rewards to rank are at least as great as the differences in the distributions of wealth of potential mates across groups. Both the shape and the magnitude of the return to rank function will depend on the fine details of the social organization within groups.

7. Our focus in this paper has been the effect of nonmarket decisions on market decisions, but there is an alternative interpretation to our results. Suppose that Bruce is interested in buying a condominium in Florida when he retires, as is everyone else in his cohort. Then as an approximation, we can imagine that the condominiums will ultimately be allocated among Bruce's cohort, with the nicest going to the wealthiest, the next nicest to the next wealthiest, and so on, with poorer people ending up with less desirable condominiums. Bruce realizes that if he is at the eighty-seventh percentile in the wealth distribution now, and he invests precisely as the others in his cohort do, he will end up in the eighty-seventh percentile condominium, regardless of how well the investments do. If

the investments soar, the price of condominiums will be bid up to absorb the higher returns, while with poor investment performance prices will drop.

This is of course an exaggeration since the supply of condominiums isn't perfectly inelastic and there are alternatives to condominiums in consumption, but there is an element of truth in the basic idea that the equilibrium prices of many inelastically supplied goods will be correlated to asset market performance. Hence, there will be less uncertainty in Bruce's final consumption if he holds the same asset portfolio as others in his cohort than if he were to hold different assets. If Bruce believes that the S&P 500 mutual fund he holds is overpriced, he can sell while others continue to hold. His relative wealth rank can change dramatically, up if Bruce is correct and down if he is incorrect. Even if the expected wealth change of selling the S&P mutual fund is positive, Bruce may rationally choose not to sell so as to avoid the risk he would face in relative rank.

This example is certainly not a tight argument, but is meant to illustrate a simple idea. Bruce's "deep" utility function has as an argument his consumption of goods and services. His consumption of goods and services will depend on his wealth. Bruce can then consider the reduced form utility function which is a concatenation of the mapping from wealth to consumption and the mapping from consumption to utility; this is standard fare in economics. We do have to bear in mind, however, that the concatenation of the two mappings is possible only as long as prices are fixed. This is generally an innocuous assumption when we analyze a single agent's investment decisions; he forecasts consumption prices with the plausible notion that they will be independent of his personal decisions.

It is a different matter, however, if we analyze a set of people rather than a single individual; we can't take prices as given independent of *all* the agents' choices. In this case, when an agent considers whether he should buy an asset, he considers the random amount of money that is associated with the asset and the (random) consumption that money will purchase. But the second step—determining the consumption that is associated with the asset's return—depends on the prices at that time. But, as we've argued, those prices depend on *other agents' asset choices*. In other words, there is a fundamental problem with a single agent trying to determine his optimal asset choice independently of other agents' choices.

The important point of this discussion is that what is sometimes taken as a primitive—an agent's utility function over wealth—is an "equilibrium" object. It is only in the context of an equilibrium of the full model with asset choices and contingent consumption choices after asset values are realized that there is an unambiguous utility function over wealth for an individual, and that utility function is appropriate only for that equilibrium. There may well be different equilibria of the full model, with different utility functions over wealth for that agents in different equilibria.[8]

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[8]We emphasize that this discussion depends on there not being complete markets. If there is a full set of contingent markets, contingent market prices will be sufficient to establish a utility function over wealth for an individual.